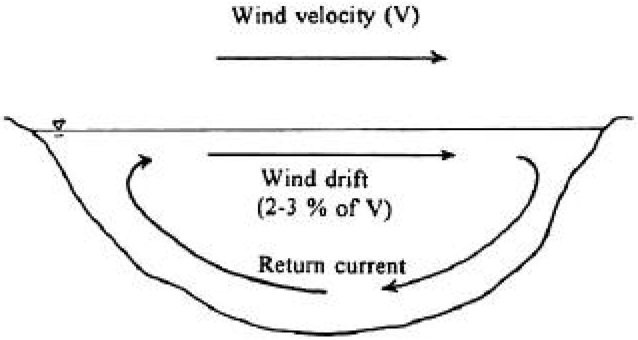
"Movement of Pollutants in Lakes"

Module 2: Surface Waters, Lecture 3

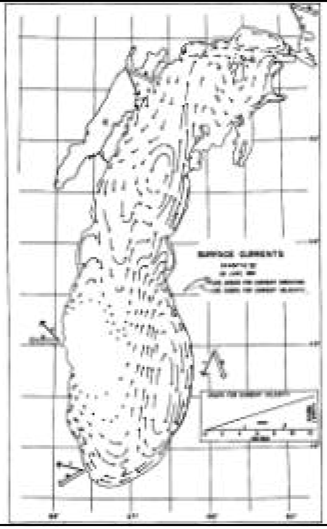
Chemical Fate and Transport in the Environment, 2nd edition. H.F. Hemond and E.J. Fechner-Levy. Academic Press. London. 2000.

Principles of Surface Water Quality Modeling and Control. R.V. Thomann and J.A. Mueller. Harper & Row, New York. 1987.

Simplest pattern of water movement in a lake:
wind-driven currents:



Hemond and Fechner-Levy 2000



Hemond and Fechner-Levy 2000

Fickian Mixing in Lakes

- A mass of tracer injected into a lake will move by advection with the water currents, but will also spread out into an ever-larger volume of water.
- Given enough time, it will tend to become completely mixed.
- This mixing is primarily due to turbulence, carrying chemicals away from regions of higher concentrations to areas of lower concentrations.

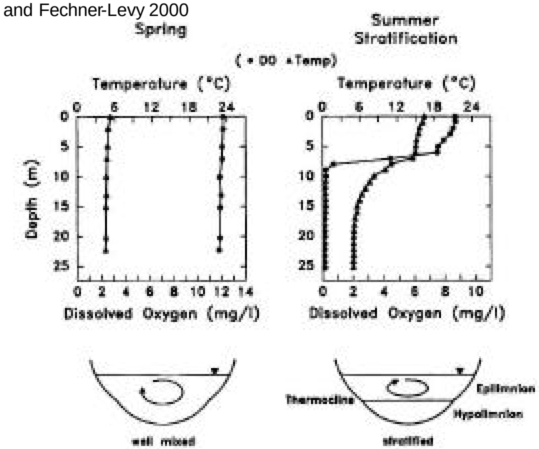
- Concentrations for an instantaneous discharge into a two-dimensional body of water (vertically mixed):

$$C(x,y,t) = \frac{M}{4\pi t\sqrt{D_x D_y}} e^{-((x-V_x t)^2/(4D_x t) + (y-V_y t)^2/4D_y t)} \bullet e^{-kt}$$

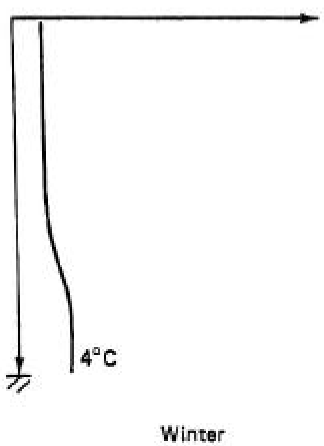
M is the mass of the chemical discharged, per depth of water [M/L]
x and y are the distances from the injection location [L]
t is the time lapsed since injection [T]
V_x and V_y are the average velocity in the x and y directions [L/T]
D_x and D_y are the Fickian transport coefficients in the x and y directions [L^2/T]
K is the first-order decay rate constant [T^{-1}]

The depth is the total depth for a vertically well-mixed lake, or the thickness of a layer in a stratified lake.

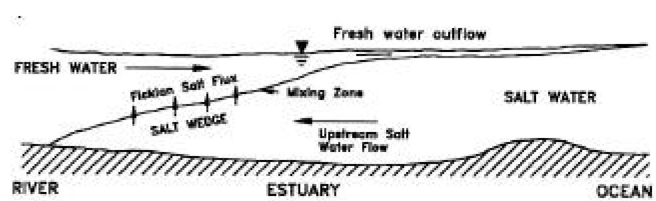
Hemond and Fechner-Levy 2000



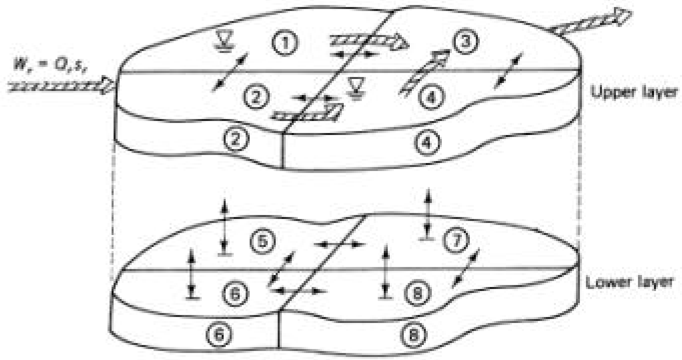
What would lake temperatures look like in winter (very cold area)?



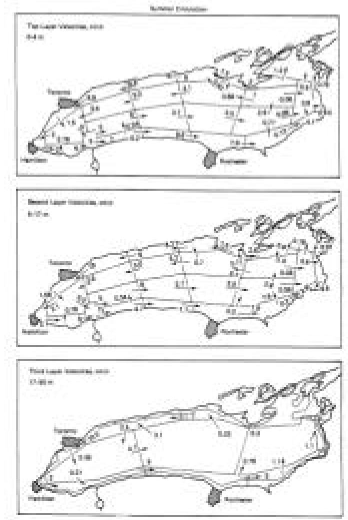
Thomann and Mueller 1987



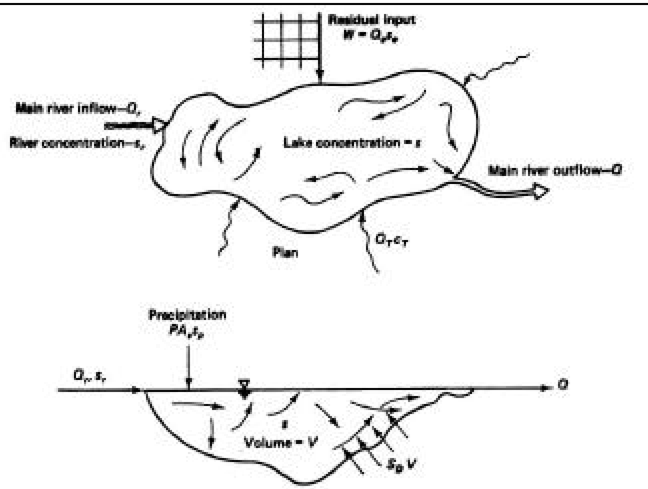
Hemond and Fechner-Levy 2000



Thomann and Mueller 1987



Thomann and Mueller 1987



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$$W = QeSe + QrSr + Q_T S_T + PAsSp + S_D V$$

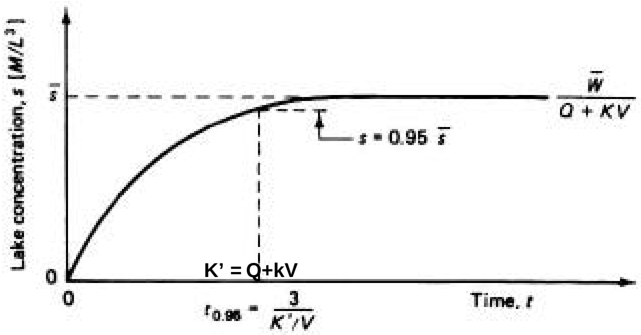
W= mass input [M/T]
QeSe = waste effluent discharged to lake
QrSr = mass from main river
QtSt = mass from tributary
PAsSp = mass input from precipitation
SdV = sediment release

Qe = effluent discharge
Qr = river flow
Qt = tributary flow

P = precipitation amount
As = lake surface area
V = lake volume

Se = effluent concentration
Sr = river concentration
St = tributary concentration
Sp = rain concentration
Sd = sediment concentration

Concentration increases with time, since the start of the discharge:



Thomann and Mueller 1987

Concentration Decrease after Discharge Stops

$$\frac{dVs}{dt} = W(t) - Qs - kVs$$

Change of mass with time =
input mass (gain) – mass outflow (loss) – decay (loss)

Assuming a constant Q and k over time

Expanding the derivative:

$$\frac{dVs}{dt} = V\frac{ds}{dt} + s\frac{dV}{dt}$$

If V is temporarily constant: $\frac{dV}{dt} = 0$

Then: $$\frac{dVs}{dt} = V\frac{ds}{dt}$$

Re-arranging results in:

$$W(t) = V\frac{ds}{dt} + Qs + kVs$$

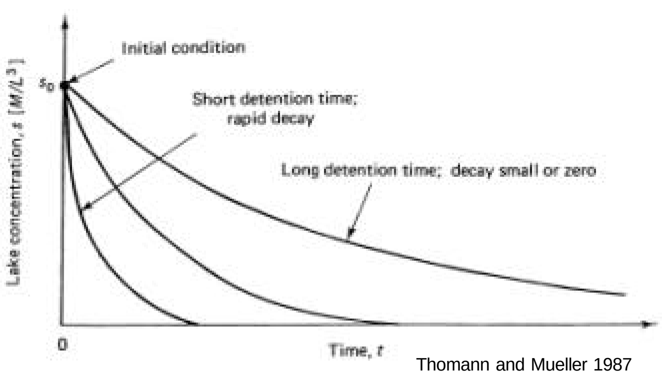
Can simplify if define: $k' = Q + kV$

Then: $$W(t) = V\frac{ds}{dt} + k's$$

Resulting in: $$s = s_0 \exp\left[-\left(\frac{1}{t_d} + k\right)t\right]$$

Where the lake detention time, td is defined as: $t_d = \frac{V}{Q}$

Concentration decreases from initial conditions (at end of input) by a combination of flushing and decay:



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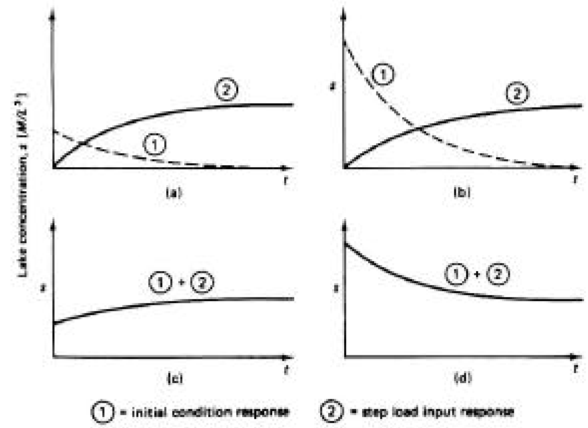
Response due to step load = sum of initial condition (from previous load), plus step load effect:

$$s = \frac{W}{Q+kV}\left\{1-\exp\left[-\left(\frac{Q}{V}+k\right)t\right]\right\} + s_o \exp\left[-\left(\frac{Q}{V}+k\right)t\right]$$

Total response and transitions can be determined by calculating individual responses and summing the effect.

Response due to initial condition and step input. (a) "Small" initial condition. (b) "Large" initial condition. (c) and (d) Sum of components.



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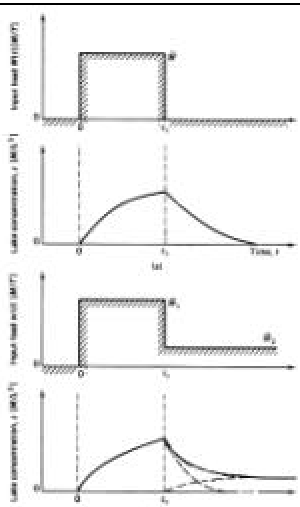
Response due to varying load:

(a) Step input and subsequent reduction to zero

(b) Step input with reduction to new load level



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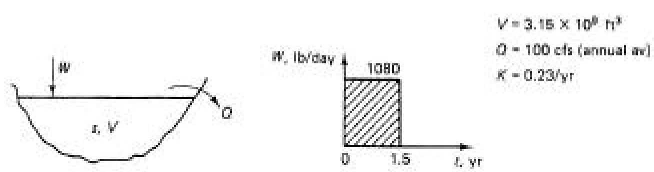
Example Problem:

A Lake (with initial s = 0), receives a load of a slowing reacti ng pesticide (triallate) of 1080 /b/day for 1.5 years and is then terminated.

1. Determine the equilibrium concentration
2. The maximum concentration
3. The time until a level of 100 μg/L is reached



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(1) Determine the equilibrium concentration

$$\bar{s} = \frac{W}{Q + kV} = \frac{W/Q}{1 + kt_d}$$

$$t_d = \frac{V}{Q} = \frac{3.15x10^9\ ft^3}{100\ ft^3/\sec} x \frac{day}{86{,}400\sec} x \frac{year}{365 days} = 1.0 year$$

$$\bar{s} = \frac{W/Q}{1 + kt_d} = \frac{\dfrac{1080 lb/day}{100\ ft^3/\sec x 86{,}400 \sec/day}}{1 + \left(\dfrac{0.23}{day}\right)(1.0 year)} = 0.000102 lb/ft^3$$

$$\bar{s} = \frac{0.000102 lb}{ft^3} x \frac{ft^3}{7.48 gal} x \frac{gal}{3.78L} x \frac{454{,}000 mg}{lb} = 1.63 mg/L = 1630 mg/L$$

(2) Determine the maximum concentration

(the max. concentration will occur at the end of the discharge time, at t=1.5 years)

$$s = \bar{s}\left\{1 - \exp\left[-(1 + kt_d)\left(\frac{t}{t_d}\right)\right]\right\}$$

$$s = 1630\ mg/L\left\{1 - \exp\left[-\left(1 + \frac{0.23}{yr} 1.0 yr\right)\left(\frac{1.5 yr}{1.0 yr}\right)\right]\right\} = 1370\ mg/L$$

Never reaches the equilibrium concentration before it starts to decrease.

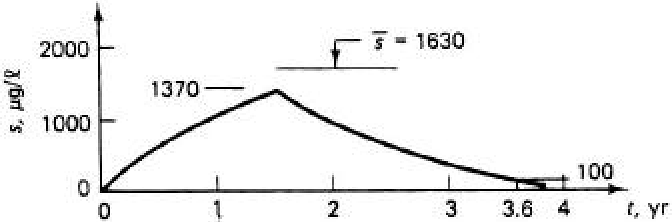
(3) Determine the time until a level of 100 μg/L is reached

$$s = s_0 \exp\left[-(1 + Kt_d)\left(\frac{t'}{t_d}\right)\right]$$

Where: $t' = t - 1.5 years$

$$100 mg/L = 1370 mg/L \exp\left[-\left(1 + \frac{0.23}{yr} 1.0 yr\right)\left(\frac{t'}{1.0 yr}\right)\right]$$

$$t' = 2.13 years$$



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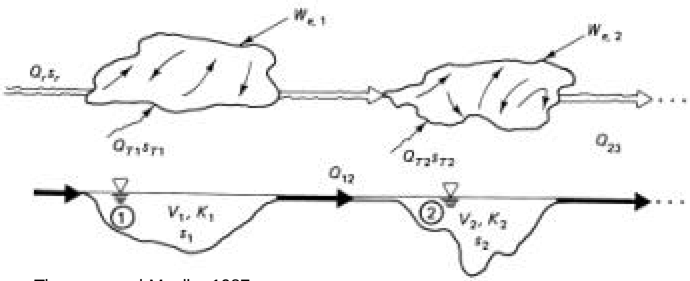

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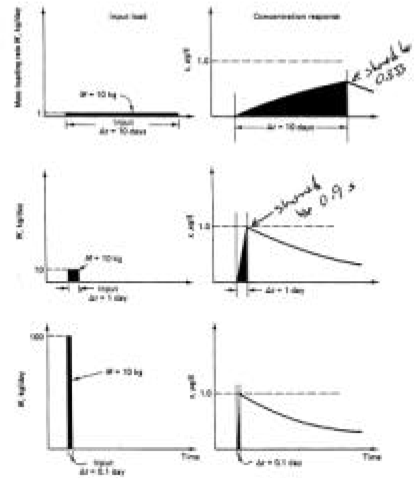

Illustration of response due to an input load discharged over various Δt time intervals ($1/t_d + K = 0.1$/day; $V = 10^{[illegible]}$ m^3).

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